

Computational Logic

Volume 1:

Classical Deductive Computing with
Classical Logic

Luis M. Augusto

© Individual author and College Publications 2018. All rights reserved.

ISBN 978-1-84890-280-1

College Publications
Scientific Director: Dov Gabbay
Managing Director: Jane Spurr

<http://www.collegepublications.co.uk>

Cover produced by Laraine Welch
Printed by Lightning Source, Milton Keynes, UK

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form, or by any means, electronic, mechanical, photocopying, recording or otherwise without prior permission, in writing, from the publisher.

Contents

Preface	xiii
I Introduction	1
0.1 Symbolic computation and classical computing	3
0.2 Logic: Formal, symbolic, deductive, and classical	5
0.3 Computational logic and its subfields	8
0.4 Classical deductive computing and its assumptions	10
II Mathematical foundations	15
1 Mathematical notions	17
1.1 Basic notions	17
1.1.1 Sets, relations, functions, and operations	17
1.1.2 Binary relations and ordered sets	24
1.2 Discrete structures	29
1.2.1 Algebras and models	29
1.2.2 Lattices	34
1.2.3 Graphs and trees	41
1.3 Mathematical induction	45
III Classical computing	49
2 Fundamentals of classical computing	51
2.1 Formal languages and grammars	51
2.1.1 Regular languages	59
2.1.2 Context-free languages	63
2.1.3 Recursively enumerable languages	76
2.1.4 The Chomsky hierarchy (I)	78
2.2 Models of computation	80
2.2.1 Finite-state machines	81
2.2.2 Pushdown automata	99
2.2.3 Turing machines	120

Contents

- 2.2.4 The Chomsky hierarchy (II) 132
- 2.3 Computability and complexity 135
 - 2.3.1 The decision problem and Turing-decidability . . . 135
 - 2.3.2 Undecidable problems and Turing-reducibility . . . 139
 - 2.3.3 The Chomsky hierarchy (III) 145
 - 2.3.4 Computational complexity 147
 - 2.3.5 The Chomsky hierarchy (IV) 163

IV Classical deduction and classical logic 165

3 Preliminaries: Formal logic, deduction, and deductive computation 167

- 3.1 Logical form I: Logical languages 168
 - 3.1.1 Alphabets, expressions, and formulae logical . . . 168
 - 3.1.2 Orders 171
 - 3.1.3 Formalization 176
- 3.2 Logical form II: Argument form 184
- 3.3 Logical meaning: Valuations and interpretations 191
- 3.4 Logical systems, logics, and logical theories 202
 - 3.4.1 Logical consequence, inference, and deduction . . . 203
 - 3.4.2 Syntactical consequence and proof theory 209
 - 3.4.3 Semantical consequence and model theory 215
 - 3.4.4 Adequateness of a deductive system 220
 - 3.4.5 Logical theories 224
- 3.5 Deductive computation 226
 - 3.5.1 Logical problems and computational solutions . . . 227
 - 3.5.2 Taming FOL undecidability 230
 - 3.5.2.1 Finite satisfiability and ground extensions 230
 - 3.5.2.2 Finite models and prefix classes 235
 - 3.5.3 The complexity of logical problems 237

4 The system CL and the logic CL 243

- 4.1 The language of classical logic 243
 - 4.1.1 The language L1 243
 - 4.1.2 Substitutions and unification for L1 245
- 4.2 Classical logical consequence 251
 - 4.2.1 Classical \heartsuit -consequences 251
 - 4.2.1.1 Classical syntactical \heartsuit -consequences . . . 252
 - 4.2.1.2 Classical semantical \heartsuit -consequences . . . 254
 - 4.2.2 Classical \blacklozenge -consequences 256
- 4.3 The logic of CL 258

4.4	Classical FO theories and the adequateness of CFOL . . .	261
4.5	The extension $CL^=$: CL with equality	269
5	Classical proofs	275
5.1	The axiom system \mathcal{L}	276
5.2	The natural deduction calculus \mathcal{NK}	279
5.3	The sequent calculus \mathcal{LK}	284
6	Classical models	291
6.1	Tarskian semantics	291
6.2	Herbrand semantics	295
6.3	Algebraic semantics: Boolean algebras	302
V	Classical deductive computing with classical logic	311
7	Classical logic and deductive computation	313
7.1	The computational problem of classical satisfiability, or SAT	314
7.2	Computerizing CFOL	321
7.2.1	Literals and clauses	322
7.2.2	Negation normal form	323
7.2.3	Prenex normal form	323
7.2.4	Skolem normal form	324
7.2.5	Conjunctive and disjunctive normal forms	325
7.3	Computing the SAT	331
7.3.1	The different forms of the SAT	331
7.3.2	The SAT and unsatisfiability I: The DPLL procedure and model finding	333
7.3.3	The SAT and unsatisfiability II: Herbrand theorem and refutation	336
8	Automated theorem proving	343
8.1	Resolution	344
8.1.1	The resolution principle for propositional logic . . .	344
8.1.2	The resolution principle for FOL	350
8.1.3	Completeness of the resolution principle	359
8.1.4	Resolution refinements	360
8.1.4.1	A-ordering	361
8.1.4.2	Hyper-resolution and semantic resolution	365
8.1.5	Paramodulation	373
8.2	Analytic tableaux	379
8.2.1	Analytic tableaux as a propositional calculus . . .	379

Contents

8.2.2	Analytic tableaux as a FO predicate calculus . . .	388
8.2.2.1	FOL tableaux without unification	389
8.2.2.2	FOL tableaux with unification	392
9	Programming	397
9.1	Logic programming as deductive programming	398
9.1.1	Query systems and programming systems	398
9.1.2	LP programs and their meaning	403
9.1.3	Resolution and LP computations	411
9.1.4	Negation as failure	422
9.2	Prolog	430
9.2.1	Prolog and Prolog	430
9.2.2	Logic + control: ! and fail	435
9.2.3	Negation in Prolog: The predicate not	441
	Bibliography	447
	Bibliographical references	449
	Index	457