

## FORMAL TRUTH TOWARD A STRONGLY FORMAL EPISTEMOLOGY

Luis M. Augusto

### Abstract

According to a widely accepted formal definition of knowledge, a statement  $p$  is known—is a fact—when it is, firstly, true, and also a justified belief. Scientific theories, as well as knowledge bases (which can coincide with the former when they have rules of the kind “if  $p$ , then  $q$ ” with a declarative interpretation), are based on facts, and thus truth is at the heart of science. Considered under a formal notion of logical consequence, theories are closed sets of facts that are then called axioms and theorems. Typically, it suffices to assert “ $p$ ”; this allows for an axiomatic approach in either classical or non-classical logics. This notwithstanding, often it is not enough simply to assert “ $p$ ”, a strong assertion that  $p$  is true being required. For this end, the truth predicate  $T(\cdot)$  to be applied to statements as  $T(p)$  appears to be appropriate. Two problems, however, are posed by this formal predication and its apparently intrinsic self-referentiality: semantic paradoxes and an infinite hierarchy of (meta)languages. The former impacts on theories by causing inconsistency, and the latter, as proven by Tarski’s Undefinability Theorem, makes any metalanguage inadequate, no matter how much richer it might be with respect to those below it in the hierarchy, giving rise to (meta)theoretical incompleteness. This state of affairs is a consequence of the displacement of the philosophical inquiry on truth from its nature to its use, a displacement that is to be attributed to A. Tarski’s work in the 1930s in a semantic definition of truth. While this approach sees truth as a definable property of statements, other approaches, in particular axiomatic theories, see truth as non-definable as a property—a position known as deflationism. This actually constitutes a major divide that I call the problem of formal truth, and which impacts on other fields of inquiry, especially so in epistemology. My starting points are as follows: 1. A formal epistemology that fully integrates strong assertions—a strongly formal epistemology—is now required, not the least because it has sooner or later to be extended to artificial autonomous knowledge agents; 2. The above divide is bridgeable by a formal notion of interpretation that both avoids semantic paradoxes and circumvents Tarski’s theorem; 3. This interpretation is to be constructed by algebraic means by seeing formal truth from the viewpoint of ordered sets and corresponding algebraic structures (e.g., lattices) and transformations (e.g., homomorphisms). The methods to be applied in this project are thus those of the history and philosophy of science allied with mathematical and philosophical logic. The construction of (a) logical system(s) in which the truth predicate has the desired properties is a major objective of this project. It is expected that this construction will be based on strong adequacy, the property of a logical system in which completeness so to say absorbs soundness.

**Key words:** Problem of formal truth; Truth paradoxes; Incompleteness results; Strong adequacy of logical systems; Algebraic semantics