Mind &/v Logic

1. Cognition

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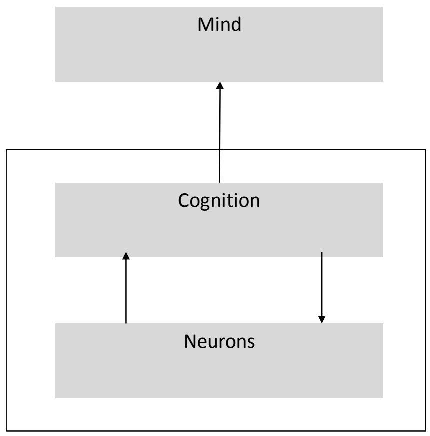
Mind

Levels of analysis



Best (1999), p. 10 (adapted from)

Cognition

- Borrowing Freud's phrase – without intending any commitment to his theory of motivation – a book like this one might be called "Stimulus Information and its Vicissitudes." As used here, the term "cognition" refers to all the processes by which the sensory input is transformed, reduced, elaborated, stored, recovered, and used. It is concerned with these processes even when they operate in the absence of relevant stimulation, as in images and hallucinations. Such terms as *sensation, perception, imagery, retention, recall, problem-solving,* and *thinking*, among many others, refer to hypothetical stages or aspects of cognition.

  Given such a sweeping definition, it is apparent that cognition is involved in everything a human being might possibly do; that every psychological phenomenon is a cognitive phenomenon.

Neisser (1967), p. 4

Thought. (Where to draw the lines?)

- By the word thought (*cogitatio*), I understand all that which so takes place in us that we of ourselves are immediately conscious of it; and, accordingly, not only to understand, to will, to imagine, but even to sense (*sentire*), are here the same as to think.

Descartes (1644/1913), p. 133 [slightly altered]

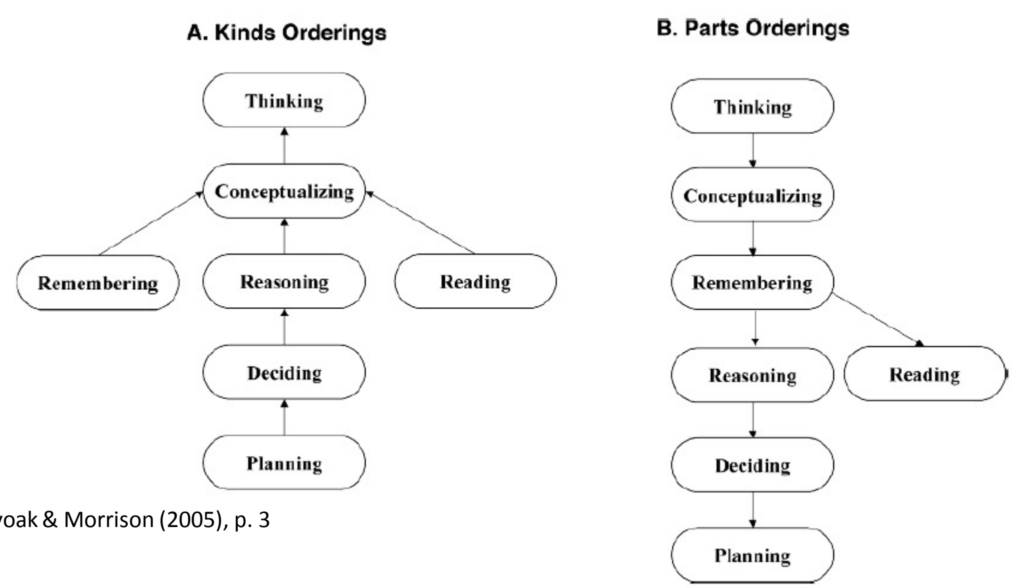
(In a letter to Husserl):

- Our thoughts are not psychic structures, and thinking is not an inner creating and constructing, but rather an apprehension of thoughts that already exist objectively.

Frege (1906 [1980, p. 41]); my transl.

- Thought is after all nothing but a substitute for a hallucinatory wish.

Freud (1900/1953), p. 567



Holyoak & Morrison (2005), p. 3

Figure 1.1. People's conceptions of the relationships among terms for mental activities. A, Ordering of "kinds." B, Ordering of "parts." (Adapted from Rips & Conrad, 1989, with permission.)

Logic

What is a logic(al system)?

- A logical system is a pair ⟨*L*, *R*⟩, where *L* is a formal language and *R* is a consequence relation, syntactic (or proof-theoretical; $\vdash$) and/or semantical (or model-theoretical; $\vDash$). In case *(strong) completeness* is verified, these consequence relations coincide, and **S** = ⟨*L*, $\vdash$⟩ defines a logical system **S**.
- The logic of *S* is the set of formulae $\Lambda_{\mathbf{S}} = \{\alpha \in L \mid \vdash_S \alpha\}$.

The language of classical logic

- SYNTAX

  - The vocabulary of the *propositional* calculus of CL is constituted by symbols for propositional parameters (*p, q, r,* …) [with or without subscripts] and the following connectives: ~ (negation), & (conjunction), ∨ (disjunction), → (material conditional), and ↔ (material equivalence).

  - If A, B are formulae, so are ~A, A→B, A∨B, A & B, A↔B.

  - In *first-order* logic (FOL), the vocabulary is augmented with symbols for variables (*x, y, z,* …), constants (*a, b, c,* …), predicates (*P, Q,* …), and quantifiers (∀, ∃).
- SEMANTICS
  - An interpretation of CL is a function *v* that assigns to each propositional parameter either TRUE or FALSE.
    - E.g.: *v*(~A) = TRUE if *v*(A) = FALSE, and FALSE otherwise.
  - Slightly (or much) more complicated for FOL, but the idea is the same.

Logical consequence

The other logical properties and relations whose recognition is central to ordinary reasoning are closely related to, and can be defined in terms of, logical consequence. We say that an argument is *valid* iff its conclusion is a logical consequence of its premises; a set of claims Γ *entails* a claim α iff α is a logical consequence of Γ; a set of claims is *consistent* iff no contradiction is a logical consequence of it, and a claim α is *independent* of a set of claims Γ iff α is not a logical consequence of Γ. Finally, a claim is a *logical truth* iff it is a logical consequence of the empty set of claims.

Blanchette (2001)

Axiomatic systems

- A consequence relation is traditionally specified by an *axiomatic system*, i.e., a set of *axiom schemata* and a set of *inference rules*, where these are expressions of the (general) form

$$\frac{\Gamma_1 \vdash \alpha_1, \ldots, \Gamma_n \vdash \alpha_n}{\Gamma \vdash \alpha}$$

An axiomatic system for CL: CLA

- 3 axiom schemata:
  - CL1: $P \to (Q \to P)$
  - CL2: $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$
  - CL3: $({\sim}P \to {\sim}Q) \to (Q \to P)$
- 1 inference rule
  - MP: $P, P \to Q \vdash Q$
- Definitions
  - $P \lor Q \coloneqq {\sim}P \to Q$
  - $P \,\&\, Q \coloneqq {\sim}(P \to {\sim}Q)$
  - $P \leftrightarrow Q \coloneqq (P \to Q) \,\&\, (Q \to P)$
- Derived rules

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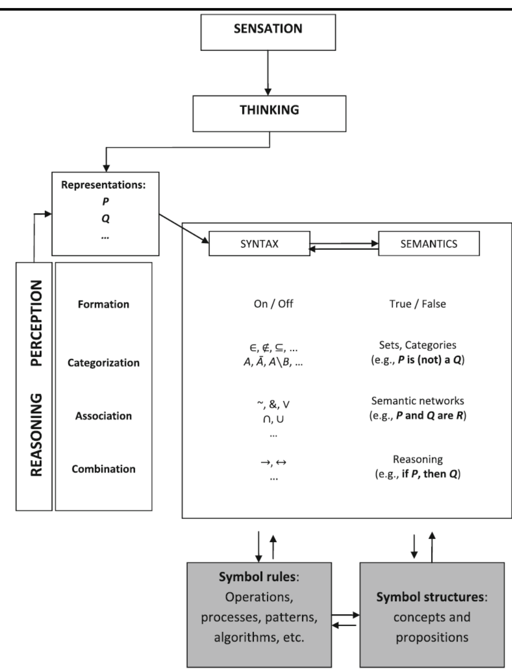

Augusto (2013)

Fig. 3 The syntax and the semantics of thought

Natural deduction

- G. Gentzen (1934/5) and S. Jaśkowski (1934)
- I intended first to set up a formal system which comes as close as possible to actual reasoning. The result was a "*calculus of natural deduction*" … [T]he essential difference between NJ-derivations and derivations in the systems of Russell, Hilbert, and Heyting is the following: In the latter systems true formulae are derived from a sequence of "basic logical formulae" by means of a few forms of inference. Natural deduction, however, does not, in general, start from basic logical propositions, but rather from assumptions to which logical deductions are applied. By means of a later inference the result is then again made independent of the assumption.

(Gentzen, 1934/5)

	Introduction Rules	**Elimination Rules**	
(1.1)	$\frac{A \quad B}{A \land B}\land I$	$\frac{A \land B}{A}\land E \qquad \frac{A \land B}{B}\land E$	(1.2)
(2.1)	$\frac{A}{A \lor B}\lor I \qquad \frac{B}{A \lor B}\lor I$	$\frac{A \lor B \quad \overset{(A)}{C} \quad \overset{(B)}{C}}{C}\lor E$	(2.2)
(3.1)	$\frac{\overset{(A)}{B}}{A \to B}\to I$	$\frac{A \quad A \to B}{B}\to E$	(3.2)
(4.1)	$\frac{\overset{(A)}{\curlywedge}}{\sim A}\sim I$	$\frac{A \quad \sim A}{\curlywedge}\sim E$	(4.2)
(5.1)	$\frac{A}{\forall x A^a_x}\forall I$	$\frac{\forall x A}{A^x_t}\forall E$	(5.2)
(6.1)	$\frac{A^x_t}{\exists x A}\exists I$	$\frac{\exists x A \quad \overset{(A^x_a)}{B}}{B}\exists E$	(6.2)

Figure 12. Inference rules of Natural Deduction.

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Mental logic (= Natural Logic)

- Logical reasoning provides some elementary information integration processing serving primarily two purposes. The first purpose is to facilitate verbal interaction by providing a set of inferences that are made automatically in processing discourse. The other purpose is to integrate information received from different sources at different times. … [A] set of inference forms coupled with a simple reasoning program for applying them to information available. The inferences of the set are very easy indeed and made essentially without error by adults. We believe that they are often made more or less automatically, and that they are understood early by children.

Braine (1990)

(1.1)	*P*; *Q* ——— *P* AND *Q*	There is a cat; There is an apple /∴ There is a cat and an apple.
(1.2)	*P* AND *Q* ——— *P*	There is a chicken and a horse /∴ There is a chicken.
(2.1)*	*P*; *Q* ——— *P* OR *Q*	There is a cat; There is an apple /∴ There is a cat or an apple.
(2.2)*	IF *P* OR *Q*, THEN *R* *P* OR *Q* ——— *R*	If there is a duck or a goose, then there is a cher There is a duck or a goose /∴ There is a cherry.
(3.1)	Suppose *P* ……… *Q* ——— IF *P* THEN *Q*	There is a fox. If there is a fox then there is a wolf. If there is a wolf then there is a lizard. There is a lizard /∴ If there is a fox then there is a lizard.
(3.2)	IF *P* THEN *Q*; *P* ——— *Q*	If there is a fox, then there is a nut; There is a fox /∴ There is a nut.
(4.1)**	*P*; INCOMPATIBLE ——— NOT *P*	There is an orange. INCOMPATIBLE / There is not an orange.
(4.2)	*P*; NOT *P* ——— INCOMPATIBLE	There is an orange; There is not an orange. / INCOMPATIBLE
(5.1)**	All *P* are *Q*. All *Q* are *R*. ǂ ——— All *P* are *R*.	All felines are cats. All cats are finicky. ǂ ∴ All felines are finicky.
(5.2)**	All *P* are *Q*. *R* is *P*. ǂ ——— *R* is *Q*.	All cats are finicky. Fritz is a cat. ǂ ∴ Fritz is finicky.
(6.1)**	*P* is *Q*. ——— Something is *Q*.	Fritz is finicky. ∴ Something is finicky.
(6.2)**	For all *P*, *P* → *Q*. ǂ Something is *P*. *R* is *P*. ǂ If *R* is *P*, then *R* is *Q*. ǂ ——— For *R*, *Q*.	All citrus fruits are bitter. ǂ Something is a citrus fruit. A lemon is a citrus fruit. ǂ If a lemon is a citrus fruit, then it is bitter. ǂ ∴ A lemon is bitter.

Figure 10. Inference schemas of Mental Logic.

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